

Solution notes for the threshold and alluvial channel design examples in NEH-654

Peter Wilcock, August 2024

(A) Threshold channel problem, NEH-654 Chapter 8, example starting on p.40.

In this problem, we seek to find the slope such that the bed material in the design channel is just at incipient motion at the design discharge. The solution in the handbook combines momentum, continuity, Manning, and the definition of the critical Shields Number to solve for critical discharge. A formula by Limerinos is used to determine Manning's n from the bed grain size. The iterative solution approach in NEH-654 starts with an initial guess and then reduces the slope until $\tau = \tau_c$ at the design discharge. The valley slope 0.007 enters the problem only as a constraint. Channel slope can be less than the valley slope (by meandering), but it is impractical to make a channel that is steeper than the valley slope!

Some questions can be raised in doing such an analysis.

1. Grain size. If the intent is that the sediment should not move at the design flow, is the median size D_{50} appropriate? Would a smaller size make sense? In general, D_{50} (of the gravel) provides a reasonable choice for estimating the critical discharge for general sediment transport, but there is scatter in such a relation (Wilcock 1993) and a risk-based approach might argue for choosing a finer size than D_{50} .
2. Roughness. Limerinos basically accounts for the grain roughness. What other factors might contribute to roughness in the channel as initially built and as it matures? Also, bank roughness will be important if the channel is relatively narrow, so an estimate of composite roughness may be needed. What happens when the total channel roughness is greater than the grain roughness provided by Limerinos?
3. Critical Shields Number. A choice of $\tau_c^* = 0.047$ is OK for the bulk sediment size distribution. If the grain size comes from a surface sample, as is commonly the case, a smaller τ_c^* will be appropriate. Whether using surface or subsurface grain size, I would recommend using the two-fraction plots for reference Shields Number τ_g^* in Wilcock and Kenworthy (2003). It is possible that τ_c^* will increase over time in a successful threshold channel, as the bed material grains jiggle and acquire physical and biological deposits in their interstices. Starting with an initial as-built values of $\tau_c^* = 0.03$ would be conservative.
4. UNCERTAINTY. One can make different reasonable choices for grain size, roughness and τ_c^* , which will lead to different estimates of the critical discharge. Because this uncertainty is real, it is important to somehow incorporate this uncertainty in one's design. One approach would be to program a Monte-Carlo analysis that determines the uncertainty in the calculated slope at the design discharge. That is, one could recalculate the problem 1000 times using grain size, roughness and τ_c^* values drawn from distributions that reflected the uncertainty in those values. An alternative would be to take the design slope as determined in the exercise and then calculate the uncertainty in the critical discharge using the design slope and specified uncertainty in grain size, roughness and τ_c^* . One could then combine the probability of failure (grain entrainment at the design flow) and the annual exceedance probability of the design flow to determine the annual probability of threshold channel failure.

The spreadsheet I developed for the problem is **NRCS_ThresholdExample.xlsx**.

The worksheet `Basic` solves the problem exactly as specified in NEH-654, although it does it more efficiently by using an iterative formulation to immediately find S . The solution in NEH-654 and repeated here uses $\tau = \rho g h S$ rather than $\tau = \rho g R S$. It is debatable whether to use h or R . R gives the total boundary stress, h is larger than R and so gives a larger stress. We might expect that the stress over the bed is larger than the mean for the entire channel, but switching h for R provides no assurance that the correct bed stress is found. A better approach is to estimate the grain stress, the portion of the total stress that acts on the bed grains.

The worksheet `Range of Widths` provides the same solution as `Basic`, but it varies the width over a wide range so you can see how the depth and slope of the design channel change with the choice of channel width. Both charts show the NRCS solution and the lower chart also shows the solution for the input specified (by you) on this worksheet.

The worksheet `GrainStress` solves the problem using a grain stress. This can be implemented by specifying the total roughness n_t and calculating the grain roughness from the bed material grain size. The grain roughness is calculated from the Manning-Strickler relation and is used with a user-specified total roughness n_t to find the grain stress τ' from the total stress τ_o . The method is explained on the spreadsheet.

(B) Alluvial channel problem, NEH-654 Chapter 9 Example 2, starting on p. 39.

The key to this problem is to first determine a transport rate and grain size for the supply reach at the design discharge, which is then used as the sediment supply to the design reach. To estimate the transport rate in the supply reach, one must first calculate the shear stress driving the transport. With a specified design discharge and an estimated transport rate and grain size, we can then develop a chart for the design reach showing the channel slope and width needed to transport the supplied sediment through channels of different width. We will follow the same logical steps as presented in NEH-654, but we'll use the Wilcock-Crowe transport model to calculate transport rate.

The spreadsheet I developed for the problem is **iSURF_NRCS_AlluvialExample.xlsm**.

(1) To get the sediment supply, I first need a value of grain stress in the supply channel. Because the roughness of the supply channel bed and banks are different, we calculate a composite roughness for the channel. The worksheet `Stress` performs the necessary calculations. The value of grain stress (expressed as a shear velocity $u^* = (\tau_o/\rho)^{1/2}$) is carried forward to calculate transport rates in the supply channel.

(2) The worksheet `SBTM`, calculates transport rate and transport grain size for a specified u_* and bed grain size. The grain size info provided in NEH-654 is incomplete for this purpose, so we define a full bed surface grain-size distribution for the supply reach that matches to given information.

(3) With the transport rate from SBTM, we now have a sediment supply (rate and grain size) to accompany the design discharge. We are ready to develop a design chart for my channel. The design discharge and sediment supply are entered into the STAB worksheet, along with the side slope angle and roughness, and press the button (Run Stab).

The STAB output is interesting. The calculated slope is remarkably close to the Brownlie solution given in NEH-654. Note also that the sediment supply rates are large: we are into the supply-sensitive part of the slope curve!